Electromagnetic phenomena 24

The multiplication of bananas by umbrellas

by Professor E. R. Laithwaite*

Adult people are very lothe to show their ignorance to their fellow men, which is a pity, because they then go around feeling inferior when, were they to declare their ignorance to a companion or small group, the odds are that nobody would know anyway! Small children are not so commonly afflicted this way. As they become teenagers they begin to have pride and to pretend, in the main, that they understand, certainly the fashionable words of the day.

So alas, it is with engineers. The older they get the more reluctant they become to say "I don't know." Just after World War II, there was a delightful film called "You know what sailors are," in which three junior naval officers in port went out drinking and had rather too much. On the way back to the ship they found an old perambulator chassis on a bombed site and used it to wheel along the most inebriated of the three. Then they saw a pawnbroker's sign—three brass balls—and climbed up a telegraph pole and took it down as a trophy (as drunks will). When they reached the ship they were slightly sobered but decided that it would be fun to sneak aboard a neighbouring ship belonging to a foreign power and mount the brass balls and pram chassis on a turntable which looked as if it should carry a radar scanner. This they did in the early hours of the morning and even managed to paint it grey to match the turntable. They returned to their ship without detection.

One of the officers in question was the junior signals officer and in the morning his immediate superior saw the erection on the foreign ship and got his binoculars on it. "What's that?" he snapped to the junior with the hang-over. Fearing discovery of their misdeeds he thought quickly and said with confidence, "It's a type 291, sir!" "A type 291?" queried his elder. "Yes sir, the very latest in radar detectors." The senior man frowned but was not going to show his ignorance by asking further questions, so his face suddenly lit up in recognition—"Of course," he said, "the 291." When the Captain spotted it, he sent for the senior signals officer and the conversation was repeated almost word for word except that the signals officer said it with even more assurance: "You know sir, the jolly old type 291." "Yes of course"—and the higher in rank it went the less likely was anyone to say "What's a type 291?" In the end the news reached the foreign powers on the ship itself who were even less likely to admit that something had got itself put there without their knowing, so they set their spies to work and the junior signals officer was tracked down as probably the only man who knew, so they kidnapped him and all his protests that it was three brass balls and a pram chassis were in vain!

So we teach our students about vector quantities—"things that have both magnitude and direction, such as velocity, force, B and H." We tell our students how to add such things—"parallelogram of vectors" and all that.

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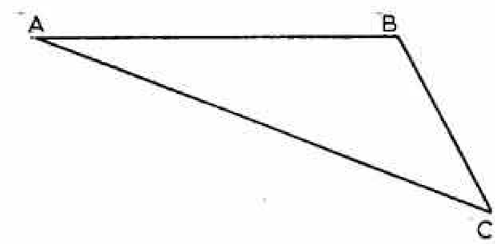

Fig. 1.—The "parallelogram" (or "triangle" in this case) of vectors is obvious if the vectors are distances

If you can add distances AB and BC and get AC, as is obvious from Fig. 1, you can travel in these directions at certain speeds so you can add velocities in the same way.

And if dx/dt, why not d^2x/dt^2? Why not indeed? Multiply the diagram by mass and you are adding forces ($M\ d^2x/dt^2$). All this is fine, but the electrical student then runs up against it. To cause forces due to electromagnetism you need to have magnetic flux density **B** *not* in the same line as an electric current which then cuts it, and experiment shows that the force thereby produced is proportional both to current and to B and to the *sine* of the single between them. To find out which way the force acts you hold up your left hand (or was it your right?) in the manner shown in Fig. 2, and repeat the magic words: "thuMb-motion, Forefinger-Field, seCond finger—Current"!

Were not we *all* reared on this rule? But to how many of us was it openly admitted, let alone stressed, that we were multiplying essentially *different* things together and getting a third thing which was not related to the first two, except in direction. Its dimensions were different—in our particular case flux density was a fiction anyway—and the rule made as much sense as saying that when you multiply bananas by umbrellas you get grandfather clocks! (except that in science at least one of the things involved is a fairy castle and never so earthy as a banana).

Some of our physics teachers might have us believe that the process we call "vector multiplication" is simple—even obvious. "You multiply a length by a length and get an area (Fig. 3a). An area is represented by a vector normal to its plane." Stuff and nonsense! In the first place you cannot tell whether the so-called "Vector" *area* is to be drawn upwards or downwards, but suppose we attempt to go into the third dimension. Length × length × length = volume (Fig. 3b), as we all know. But let us do it by vectors. AB × BC = OP (so some of us are told). If that be so, then OP × BD = O, for the two are parallel. Therefore all volumes are zero! Let's face it, distances do not now look like true vectors, yet when we refer to distances walked along the ground they must be added like velocities and forces, which *are* true vectors.

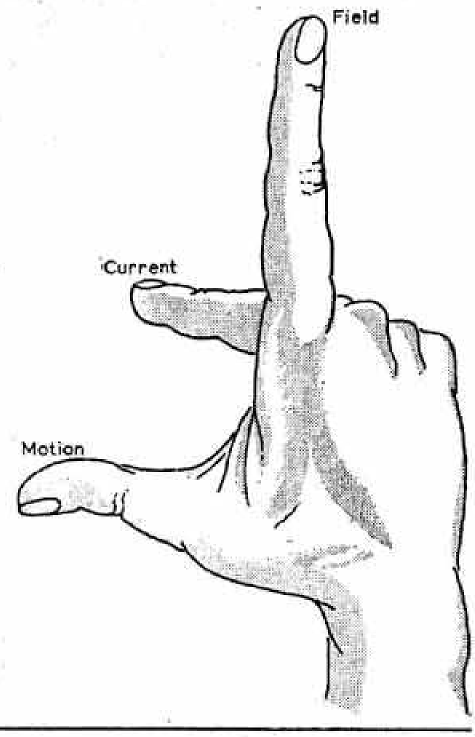

Fig. 2.—The "sacred" right hand rule

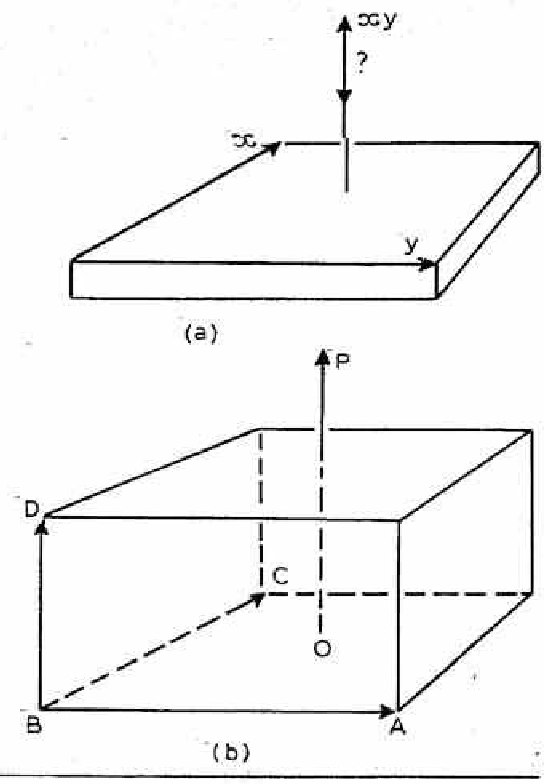

Fig. 3.—(a) Multiplication of distance by distance appears to be consistent with the rules of vector multiplication, but (b) a triple multiplication of distances does not

The solution to this apparent paradox lies in a truth which I was never told as a pupil or as a student. In this game of multiplying bananas by umbrellas, you are *not* allowed to multiply bananas by bananas, nor umbrellas by umbrellas. It is obvious nonsense anyway. Common sense tells us we can only *add* bananas to bananas. Why we cannot multiply them is a ludicrous question. So why can we multiply bananas by umbrellas, and therefore flux density by electric current? Why did no one tell me early in my career that the vector products "velocity × velocity" and "force × force" are meaningless? and what is worse, even quantities like force, acceleration, current and flux cannot be taken in pairs at random and multiplied vectorially or the answer may be as meaningless as trying to multiply beef steak by physical exercise. Indeed the latter product might even be said to make more sense as if we wrote:

Beef steak × physical exercise = athletics

I have no doubt that if this equation were to be "put across" in the classroom with as much conviction as are Fleming's Left and Right Hand Rules the children would have no difficulty in swallowing it. But they could not digest it, and I suggest that we have never digested vector multiplication, for if we had, we would not continue to regard gyroscopes as "very difficult to understand," which is the reaction of most engineers when asked what they know about these curious devices.

The more learned and those preferring the mathematical approach to their engineering will probably reply. "Their action can all be understood by the Conservation of Momentum," with the same degree of conviction as "The Type 291." For there is more to gyroscopes than mere vector multiplication and the extension of the linear application of Newton's Laws which gives the equation:

$$\text{Force} = \text{mass} \times \text{acceleration} \qquad (1)$$

to the rotary case of:

$$\text{Torque} = \text{Moment of inertia} \times \text{angular acceleration} \qquad (2)$$

for equation (1) has effectively been multiplied throughout by length, which may or may not remain a constant as a mass revolves, as for example, if it consisted of a deformable material.

Before we venture into the uncharted seas of the gyro, let us look at the first occasion on which the wool was well and truly pulled over our eyes on the subject of applied mathematics, as taught at "A" level. "Circular Motion," I believe, is the heading. Centrifugal force (or should it be "centripetal"?) is about to be derived, and I have heard many pedants argue on which name should be retained. Whichever you prefer, it is the thing that keeps the string in tension when you whirl a chestnut around on the end of it. To refresh your memories, the "proof" goes like this:

Consider a wheel (Fig. 4a) to be rotating at constant angular velocity ω. Instantaneously, the point A is travelling at velocity v_1 (and v_1 is a vector). At an instant dt later, the point A is displaced by angle $d\Theta$, and its velocity, although of the same magnitude, is now v_2. To find the *change* in velocity, subtract v_1 from v_2 (vectorially, of course) and this has been done on Fig. 4a by adding $(-v_1)$ (dotted) to v_2 by the parallelogram of vectors. So the resultant, dv, is seen to point towards the centre of rotation and by simple geometry is of size $dv = v\, d\Theta$.

Thus:

$$\text{Acceleration} = \frac{dv}{dt} = v\frac{d\Theta}{dt} = v\omega = r\omega^2 = v^2/r.$$

Here the narrative stops, for it makes an ideal "A" level question. To enquire further would be to extend the subject beyond what the teacher was taught, or what the writers of textbooks were taught and if you think I am being unrealistic when I say that the "A" level exam. is an end in itself, you must remember that I was once Chairman of a Physics Advisory Committee for one of the Examining Boards. We were trying to shorten the syllabus and my blunt remark that "The magnetometer can go, for a start," was greeted by an eerie silence in which all looked to the

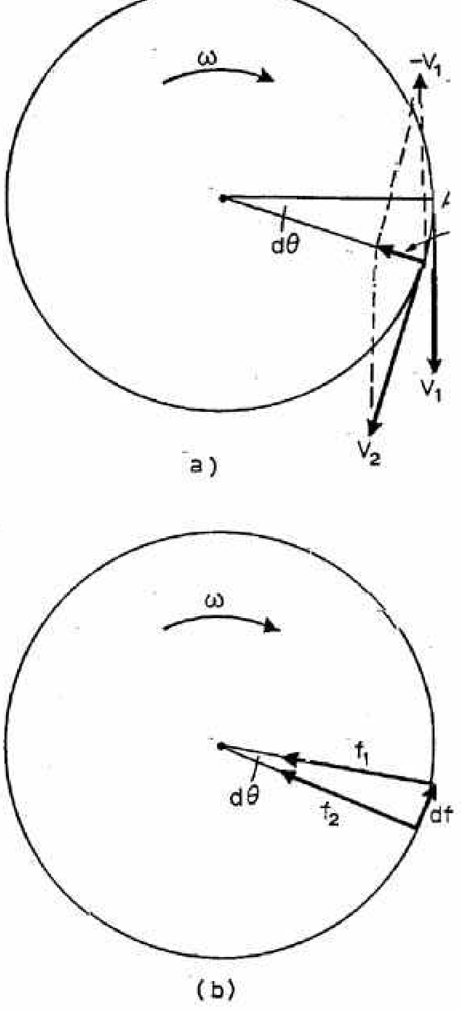

Fig. 4.—(a) The standard method of proving the existence of radial acceleration in a rotating mass. (b) The same rule extended to prove the apparent tangential rate of change of acceleration

Chief Examiner to oppose this heresy. "With respect, Professor," he said, "you can't remove the magnetometer, or we'll have nothing in magnetism to set for the practical." The year?—1966, I believe!

Let us continue our vector argument on Fig. 4a, for the acceleration we have derived is itself a vector, so [illegible] treated similarly as in Fig. 4b. For a time interval dt, and rotation $d\Theta$, f_1 becomes f_2 and df (tangentially [illegible] wards!) is equal to $f\,d\Theta$, so

$$\frac{df}{dt} = f\,\frac{d\Theta}{dt} = f\omega = r\omega^3.$$

This rate of change of acceleration is not a *small* quantity. It is ω times the centripetal acceleration. But how shall we treat it? Does it give rise to a force? If so, what is the constant of proportionality which must have the dimensions (mass × time)? What did Newton or [illegible] followers have to say on the subject of rate of change of acceleration?

The answers to some of these questions and the generation of some more must wait until my next article where the gyroscope will be seen to make such a mess of fundamental mechanics that the multiplication of bananas by umbrellas will seem a most logical thing to do! (see Fig. 5).

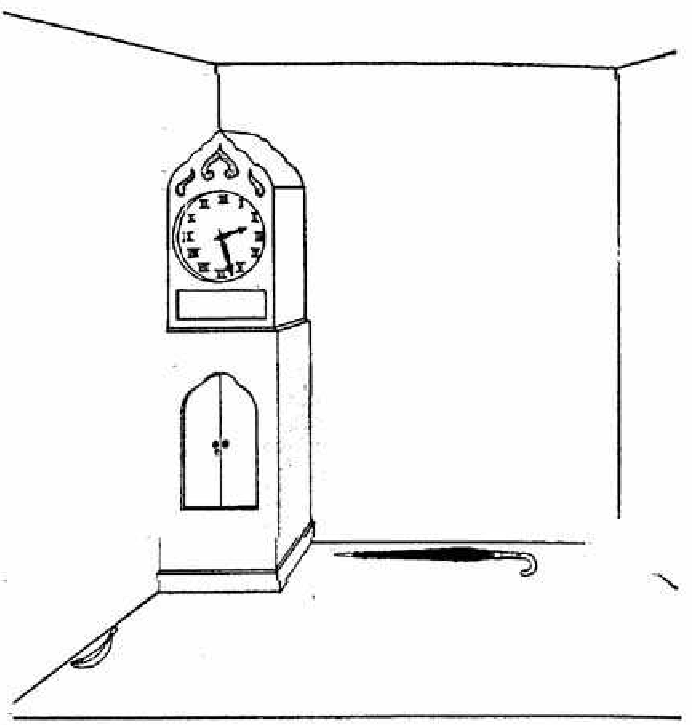

Fig. 5.—As every good engineer knows, the result of multiplying a banana by an umbrella cannot ever be bananas or umbrellas

book

Practical tips for managers

What to do and what not to do to make life easier for yourself at work. By Al Kelly. McGraw Hill. Pp. 101; figs. £1·95.

Many books have been written on management; much time and money is spent in training managers; and yet the standards of management in this country (and in many others) still often leaves a lot to be desired. Can it be that much of modern management training assumes that many of the fundamentals are so obvious as to not need covering? Or are the basic requirements not really so obvious after all?

Al Kelly's book is short, but is written with humour. Mr Kelly shows that the first job of a manager is to apply his art to his own work, and his own relationship with the people with whom he must deal. Much of the text is a lesson in good manners—an old-fashioned virtue which, alas, many modern managers seem to consider a vice.

Mr Kelly, project engineer with the Electricity Supply Board in Dublin, has been able to put his theories to the test. He wants to dispense with much of the secrecy and humbug which still surrounds many of the traditional management attitudes. He criticises many of the "traditional" trappings of management—taking work home, for example; overdoing correspondence which is "private and confidential"; pyramidical management structures; and many others. On the other hand, he has short shrift for incompetents—whether the subordinates or the boss.

Among the works he suggests the reader should also consult are those of Parkinson (of *Parkinson's Law*) and Dr Peter (of *The Peter Principle*). This book leads the reader to consider himself and his own career critically, and to recognise his own shortcomings rather than giving him an excuse to laugh at others. If the reader learns to approach his own work in the spirit in which this book was written, Mr Kelly will have achieved most of his objectives.—RAK.